Minisymposium on

Topics Related to Hilbert’s 16\textsuperscript{th} Problem

Center for Dynamical Systems and Nonlinear Studies
School of Mathematics
Georgia Institute of Technology

Monday, April 29, 2002

Organizers:
Shui-Nee Chow and Konstantin Mischaikow

Speakers:
Freddy Dumortier  Limburgs Universitair Centrum, Belgium
Maoan Han  Shanghai Jiaotong University, China
Chengzhi Li  Peking University, China
Christiane Rousseau  University of Montreal, Canada
Huaaiping Zhu  Georgia Institute of Technology, USA

Time:
9:50-11:55 am, 2:05-4:55 pm; Tuesday, April 29, 2002.

Place: Skiles 269

ALL ARE CORDIALLY WELCOME
Program of the Minisymposium on
Topics Related to Hilbert’s 16th Problem
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9:50-10:00 Opening: Shui-Nee Chow
10:05-10:55 Christiane Rousseau
University of Montreal, Canada
11:05-11:55 Maoan Han
Shanghai Jiaotong University, China
12:00-14:00 Lunch
14:05-14:55 Freddy Dumortier
Limburgs Universitair Centrum, Belgium
15:05-15:55 Huaiping Zhu
Georgia Institute of Technology
16:05-16:55 Chengzhi Li
Peking University, China

* Beverages will be served during the breaks

TITLE AND ABSTRACTS

10:05-10:55 C. ROUSSEAU: Hilbert’s 16th Problem, Finite Cyclicitiy of Graph- ics and Analytic Normal Forms for Singularities of Analytic Vector Fields.

Abstract: Hilbert’s 16th problem deals with the existence of a uniform bound for the number of limit cycles of polynomial systems of a given degree. One approach to this problem is to study the finite cyclicity of graphics appearing in analytic families of vector fields having compact phase space and compact parameter space. In studying the finite cyclicity of graphics one brings the singular points of the graphics to normal form. For several generic graphics it is sufficient to use polynomial normal forms in class \( C^n \). However other graphics require finer normal forms in analytic class. The changes of coordinates to normal forms generically diverge. We explore why. Moreover normal forms for families of vector fields are used to classify equivalence classes of conjugate (or equivalent) analytic families of vector fields in the neighborhood of singular points. We expose our first results on the analytic classification of families unfolding a saddle-node. This is done through the analytic classification of germs of analytic families unfolding a complex analytic diffeomorphisms of the complex plane with a double fixed point at the origin.


Abstract: Suppose a central symmetry \( C^\infty \) planar system has a double homoclinic loop \( L \) with a saddle point at the origin. Let \( dvi \) denote the divergence of the system. It
is well known that if $\text{dvi}(0) < 0$ ($> 0$) then $L$ is both inner and outer stable (unstable).
Recently we found that if $\text{div}(0) = \int_L \text{div} dt = 0$ and the first order saddle value at the
origin is not zero, then the inner and outer stability of $L$ is different. In this paper we
study the inner and outer stability of $L$ in more details. By using normal form theory
and Poincaré map, we give two series of constants for determining its inner and outer
stability. These two series of constants are closely related to each other.

Abstract: The talk is based on joint work with R. Roussarie. It deals with generic
perturbations from a Hamiltonian planar vector field and more precisely with the num-
ber and bifurcation pattern of the limit cycles. Near regular closed orbits and near
saddle loops the description completely relies on the analogous study of the zeros of the
related Abelian integral. We show that near a 2-saddle cycle, the number of limit cycles
produced in unfoldings with one unbroken connection, can exceed the number of zeros
of the related Abelian integral, even if the latter represents a stable elementary catas-
trophy. We however also show that in general, finite codimension of the Abelian integral
leads to a finite upper bound on the local cyclicity. In the treatment, we introduce the
notion of simple asymptotic scale deformation.

15:05-15:55 H. ZHU: Finite cyclicity of degenerate graphics and finiteness
part of Hilbert’s $16^{th}$ problem
Abstract: A graphic (singular cycle, limit periodic set, polycycle) of a planar vector
field is an invariant set of the vector field involving regular orbits and singular points.
The degenerate graphics are the graphics with a nilpotent singular point or a line (curve)
of singular points. The question of finding the number of limit cycles which appear by
perturbation of a graphic in a generic family and the problem of finite cyclicity is closely
related to Hilbert-Arnold Problem and Hilbert’s $16^{th}$ problem. I will talk about the
finite cyclicity of the degenerate graphics with a nilpotent saddle or elliptic point. The
main tools include the normal form theory and “global blow-up” techniques. As an
application, I will also discuss the limit cycles and a degenerate graphic appearing in a
predator-prey model.

16:05-16:55 C. LI: A Uniform Proof of the Weak Hilbert’s $16^{th}$ Problem for
$n = 2$.
Abstract: The weak Hilbert’s $16^{th}$ problem is asking for a least upper bound of
the number of zeros of Abelian integrals for all polynomial 1-forms of degree $n$ over all
compact algebraic curves of degree $n + 1$. This number is closely related to the number
of limit cycles for polynomial differential systems of degree $n$ which are perturbations
of polynomial Hamiltonians systems of degree $n$. For $n = 2$, this problem was solved by
Horozov and Iliev (’94), Zhang and Li (’97), Gavrilov (’01), and Li and Zhang (preprint)
for different topological structures of the unperturbed Hamiltonian systems. In a joint
work with F. Chen we give a uniform proof for these 5 cases by using some geometric
approach and deformation argument.