Spectral approximation of piecewise analytic functions

While classical wavelet analysis is adequate for a characterization of local Besov spaces, we propose a polynomial frame on the unit interval adequate for a characterization of functions analytic at a point on the interval. Thus, \textit{at each point} on the interval, the behavior of the coefficients in our frame expansion can be used to detect whether the function is analytic at that point or not. The corresponding approximation operators yield an exponentially decreasing rate of approximation in the vicinity of points of analyticity and a near best approximation on the whole interval. In spite of this high localization, the construction of our operators are based on the (globally defined) Fourier coefficients in a general orthogonal polynomial expansion. Previously known results in this direction utilize Chebyshev coefficients, and the techniques to obtain these cannot be used for a similar study of general orthogonal polynomial systems. Another novelty of our paper is that while all the previous estimates for localization of polynomial kernels known to us are deduced using such special function properties of the orthogonal polynomials as asymptotics or explicit formulas for the Christoffel–Darboux kernel, we suggest a very simple idea to obtain exponentially localized kernels based on a general system of orthogonal polynomials, for which the Cesàro means of some order are uniformly bounded. The boundedness of these means is known in a number of cases, where no special function properties are known. The talk is based on joint work with J. Prestin and F. Filbir.

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